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RESEARCH ARTICLE

BEHAVIOUR OF A-SYMPLECTOMORPHISMS ON THE RICCI A-TENSORS

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ABSTRACT

In this paper, we define the curvature A-tensor of a symplectic A-connection and the curvature operator associated with a symplectic Yang-Mills field. We show that the symplectic curvature A-tensor, the Ricci curvature A-tensor and the symplectic Ricci A-tensor are invariant under the group sheaf of A-symplectomorphisms. Finally we introduce on a symplectic vector sheaf the Euler-Langrage equation for the symplectic Yang-Mills functional.

Key words: Symplectic Yang-Mills field, Symplectic A-Connection, A-Symplectomorphism, Curvature A-Tensor of An A-connection.

INTRODUCTION

Habermann et al., 2007 p.7) generalized the symplectic Ricci operator of connections on a C^∞ manifold to connections in a vector bundle. Based on the fact that geometry of vector sheaves is the abstract analog of geometry of vector bundles, working with A-connections in a symplectic vector sheaf (E, σ) i.e a loccally free A-module endowed with a symplectic A-form, we define the curvature A-tensor of this A-connection and the curvature operator associated with this symplectic Yang-Mills field {(E,∇), σ} which allow us to introduce the Ricci curvature A-tensor, the symplectic curvature A-tensor, the symplectic Ricci operator and the symplectic Ricci A-tensor. Next, we examine the behaviour of the curvature A-tensors under the group sheaf of A-symplectomorphisms. In (Vaisman, 1985), I.Vaisman gave the *sp(n)*-Decomposition of the symplectic curvature tensor associated to the curvature of a symplectic connection on a given symplectic manifold into two irreducible components under the action of the symplectic group on the space of symplectic curvature tensors. On the same line, under the action of the group sheaf of A-symplectomorphisms on the space of symplectic curvature A-tensors of (E, σ), we decompose it into *Sp* E-irreducible components. The Author in (Boubel, 2003) claimed that on a Riemannian manifold (M, g) there exists an unique connection ∇ such that $\nabla g = 0$ called the Levi-Civita connection. On a given a symplectic manifold (M, σ), the set of symplectic connections ∇, i.e $\nabla\sigma = 0$, is an infinite dimensional affine space. In order to select the so-called preferred symplectic connections, Bourgeois and Cahen introduced a variational principle (Bourgeois et al., 1999).

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The fundamental concepts presented in this paper are based on the classical ones (see Bieliavsky (2006), Cahen (2000), Gutt (1998) and Habermann (Habermann, 2007).Unless otherwise mentioned, throught this paper A ≡ (A, τ, X), is a sheaf of commutative, associative and unital $\mathbb{C}$-algebras over a topological space X and the triplet (A, ∂, E) is a differential triad.

SYMPLECTIC RICCI A-TENSORS

We recall that for a given symplectic vector sheaf (E, σ) on a topological space X,

$\otimes^k E^* = Hom_A(\otimes^k E, A)$ is the sheaf of covariant A-tensors of order k over E. We notice that the sheaf of covariant A - tensors of order k over E can be written as follows :

$$\otimes^k E^* = \otimes^0 E \otimes_A \otimes^k E^*$$

$$= Hom_A(\otimes^k E, A).$$

Definition 2.1 Let {(E, ∇), σ} be a symplectic vector sheaf on X i.e a locally free A-module of finite rank 2n over X endowed with a symplectic A-connection ∇ and a symplectic A-form σ,the curvature A-tensor of ∇ relative to a local gauge U of E is defined by

$$R_{|U}(s, t)r = R(s, t)r = (\nabla(s)\nabla(t) - \nabla(t)\nabla(s) - \nabla([s, t]))r \quad (1)$$

for any s, t, r in E(U) and U an open subset of X where R is the curvature of the A-connection ∇.

As $\nabla \in Conn_A(E, \sigma)$ it follows that

$$\sigma(R(s, t)r, l) + \sigma(r, R(s, t)l) = 0 \qquad (2)$$

for any s, t, r, l in E(U) and U an open subset of X.

The Lie product appeared above is defined as follows: for a given A-module E on X, (A, ∂, Ω) a differential triad and $e^U = \{U, e_1, e_2, ..., e_{2n}\}$ a local gauge of E where U is an open subset of X,

$$[s, t] = \sum_{i=1}^{2n}\left(\sum_{j=1}^{2n}(t_j\, \partial(s_i) - s_j\, \partial(t_i))(e_j)\right)e_i \qquad (3)$$

is the Lie product of sections s and t.

Note that, for any i and j ∈ I = {1, 2, ..., 2n},

$(e_i, e_j) = ((1_A\, \partial(1_A) \quad 1_A\, \partial(1_A))(e_j))e_i = 0.$

Using the Lie product of sections defined above, one gets

$$\begin{aligned} R_{|U}(s, t)r &= R(s, t)r \\ &= (\ \ (t)\ \ (s) - \ \ (s)\ \ (t) - \ \ ((t, s)))r \\ &= \ \ R(t, s)r \\ &= \ \ R_{|U}(t, s)r \end{aligned} \qquad (4)$$

for any s, t, r in E(U) and U an open subset of X.

The two Bianchi identities established in the Classical Differential Geometry (CDG) hold in Abstract Differential Geometry (ADG). For any sections s, t, r in E(U) and U an open subset of X, one gets respectively the first and the second Bianchi identities :

$$R_{|U}(s, t)r + R_{|U}(t, r)s + R_{|U}(r, s)t = 0 \qquad (5)$$

$$(\ \ (s)R)(t, r) + (\ \ (t)R)(r, s) + (\ \ (r)R)(s, t) = 0. \qquad (6)$$

Definition 2.2 Let (E, σ) be a symplectic vector sheaf on X, the curvature operator associated with the symplectic Yang-Mills field (E,) relative to a local gauge U of E is defined as

$$R_{|U}(.,s)t = R(.,s)t \in (End_A)(U) \qquad (7)$$

for any s, t in E(U) and U an open subset of X.

Since R (., s)t ∈ $(End_A E)(U)$ and E is a locally free A-module of finite rank 2n over X i.e for any open subset U of X, $E_{|U}$ $A^{2n}{}_{|U} = (A_{|U})^{2n}$, one obtains

$R(., s)t \in Hom_{A|U}(E(U), E(U)) = Hom_A(A(U)^{2n}, A(U)^{2n}) = M_{2n}(A(U))$, see (7, p.150).

Definition 2.3 Let {(E,), σ} be a symplectic Yang-Mills field on X, the Ricci curvature A-tensor of , denoted ric, is defined for a local gauge U of E as the trace of the matrix R(., s)t with s, t in E(U) and U an open subset of X.

For any sections s, t, r of E(U) and open subset U of X,

$$ric_{|U}(s, t) = ric(s, t) = tr\,(r \rightarrow R(r, s)\,t). \qquad (8)$$

The Ricci curvature A-tensor is a covariant A-tensor of order 2 over E i.e

$ric \in {}^2E^* = {}^0E \ \ _A \ \ {}^2E^* = Hom_A(\ {}^2E, A).$

Definition 2.4 Let E be a vector sheaf on a topological space X, J ∈ End_A E so that $J^2 = \ \ id_E$ is called an A-complex structure on E.

Consider {(E,), σ} a symplectic Yang-Mills field on X of rank 2n. For a given local gauge of $e^U = \{U; e_1, e_2, ..., e_{2n}\}$ of E, J ∈ End_A E such that $J_{|U}(e_i) = e_{n+i}$, for any i=1,...,n, is an A-complex structure on E with U a open subset of X. For the classical case, see (Habermann et al., 2007) page 6. Under the previous consideration, the Ricci curvature A-tensor of can be written as follows

$$ric_{|U}(s, t) = ric(s, t) = \sum_{i=1}^{2n}\sigma(R_{|U}(e_i, s)t, J_{|U}(e_i)). \qquad (9)$$

Definition 2.5 Let {(E,), σ} be a symplectic Yang-Mills field on X, the symplectic curvature A-tensor associated to the curvature of a symplectic A-connection , denoted sR, is defined for a local gauge U of E by

$$sR_{|U}(s, t, r, l) = sR(s, t, r, l) = \sigma(R_{|U}(s, t)r, l) \qquad (10)$$

with s, t, r, l in E(U) and an open subset U of X.

sR is a covariant A-tensor of order 4 over E i.e $sR \in {}^4E = {}^0E \ \ _A \ \ {}^4E = Hom_A(\ {}^4E, A)$.

Proposition 2.6 Let {(E,), σ} be a symplectic Yang-Mills field on X, the symplectic curvature A-tensor sR satisfies the following conditions :

$$sR(s, t, r, l) = \ \ sR(t, s, r, l), \qquad (11)$$

$$sR(s, t, r, l) = sR(s, t, l, r), \qquad (12)$$

for any s, t, r, l ∈ E(U) and an open subset U of X.

Proof

From the definition of the symplectic curvature A-tensor and since R(s, t) = R(t, s), we get $sR_{|U}(s, t, r, l) = sR(s, t, r, l) = \sigma(R(s, t)r, l) = \ \ \sigma(R(t, s)r, l) = \ \ sR(t, s, r, l)$ for any s, t ∈ E(U) and an open subset U of X.

Since σ(R(s, t)r, l) + σ(r, R(s, t)l) = 0 for a symplectic A-connection and using again the fact that σ is a skew-symmetric A-form, we get

$\sigma(R(s, t)r, l) = \ \ \sigma(r, R(s, t)l)$
$= \sigma(R(s, t)l, r)$

for any s, t, r, l ∈ E(U) and an open subset U of X.

Thus, sR(s, t, r, l) = sR(s, t, l, r).

Proposition 2.7 Let {(E,), σ} be a symplectic Yang-Mills field on X, the symplectic curvature A-tensor sR satisfies the following condition:

$$sR(s, t, r, l) + sR(t, r, s, l) + sR(r, s, t, l) = 0, \qquad (13)$$

for any s, t, r, l ∈ E(U) and an open subset U of X.

Proof. Using the definition of the symplectic curvature A-tensor and by means of the first Bianchi identity, we derive for any s, t, r, l ∈ E(U) and an open subset U of X

sR(s, t, r, l) + sR(t, r, s, l) + sR(r, s, t, l) = σ(R(s, t)r, l) + σ(R(t, r)s, l) + σ(R(r, s)t, l)

= σ(R(s, t)r + R(t, r)s + R(r, s)t, l)
= σ(0, l)
=0

where we have used again the fact that σ is A-linear in its first component.

Definition 2.8 Let {(E, ∇), σ} be a symplectic Yang-Mills field on X, the symplectic Ricci operator, denoted sRic, is defined for a local gauge U of E as a section of End_A (U) such that

$sRic_{|U}$ (s) = sRic(s) = $\sum_{i=1}^{n} R_{|U}$ (ei , $J_{|U}$ (ei)s (14)

with s in E(U) and an open subset U of X.

Definition 2.9 Let {(E, ∇), σ} be a symplectic Yang-Mills field on X, the symplectic Ricci A-tensor, denoted sric, is defined for a local gauge U of E by

$sric_{|U}$ (s, t) = sric(s, t) = σ(sRic(s), t) (15)

with s, t in E(U) and an open subset U of X.

sric is a covariant A-tensor of order 2 over E i.e sric ∈ ^{2}E = ^{0}E $_A$ ^{2}E = Hom_A(^{2}E, A).

Proposition 2.10 Let (E, σ) be a symplectic vector vector sheaf, if $Conn_A$ (E, σ) then $sric_{|U}$ (s, t) = $sric_{|U}$ (t, s) for any s, t ∈ E(U) and an open subset U of X.

Proof. Let e^U = {U; e_1, e_2, ..., e_{2n}} be a local gauge of E, for any s, t ∈ E(U) and an open subset U of X,

$sric_{|U}$ (s, t) = sric(s, t)
= σ(sRic(s), t)
= σ($\sum_{i=1}^{n} R$(e_i, J (e_i)s, t)
= σ($\sum_{i=1}^{n} R$(e_i, J (e_i)t, s)
= σ(sRic(t), s)
= sric(t, s)
= $sric_{|U}$ (t, s).

ACTION OF THE GROUP SHEAF SpE ON SYMPLECTIC RICCI A-TENSOR

We first recall that under the action of the group sheaf of symplectomorphisms of E, the curvature R = R( ) of a symplectic A-connection becomes R' = R(') = φ R) $φ^{-1}$. Using the previous consideration, we assume that for any φ ∈ SpE

$R_{|U}$ (φ(s), φ(t))φ(r) = R (φ(s), φ(t))φ(r)

= (φ R $φ^{-1}$) (φ(s), φ(t))φ(r) (16)

with s, t, r in E(U) and U an open subset of X.

Consider φ ∈ SpE, since the curvature operator $R_{|U}$ (., s)t ∈ $End_{A(U)}$ E(U) then $R'_{|U}$ (., φ(s))φ(t) belongs

to $End_{A(U)}$ E(U) and R' (., φ(s))φ(t) φ = φ R(., s)t. Thus, R' (., φ(s))φ(t) = φ R(., s)t $φ^{-1}$.

For any r in E(U) and U an open subset of X, (R' (., φ(s))φ(t))(φ(r)) = (φ R(., s)t $φ^{-1}$)(φ(r)) and

R'(φ(r), φ(s))φ(t) = (φ R(., s)t)($φ^{-1}$ (φ(r)))
= (φ R(., s)t)(r)
= φ R(r, s)t
= φ(R(r, s)t).

Proposition 3.1 Let {(E, ∇), σ} be a symplectic Yang-Mills field on X, the symplectic curvature A-tensor and the ricci curvature A-tensor are invariant under the action of the group sheaf of symplectomorphisms of E i.e

$sR'_{|U}$ (φ(s), φ(t), φ(r), φ(l)) = $sR_{|U}$ (s, t, r, l),

$ric'_{|U}$ (φ(s), φ(t)) = $ric_{|U}$ (s, t)

for any s, t, r, l ∈ E(U) and an open subset U of X.

Proof.

It follows from the two relations R'(φ(s), φ(t)) φ(r) = φ(R(s, t)r) and σ (φ , φ) = σ that

sR'(φ(s), φ(t), φ(r), φ(l)) = σ(R(φ(s), φ(t)) φ(r), φ(l))
= σ(φ(R(s, t)r), φ(l))
= σ(R(s, t)r, l)
= sR(s, t, r, l).

for any s, t, r, l ∈ E(U) and an open subset U of X.

Let e^U = {U; e_1, e_2, ..., e_{2n}} be a local gauge of E and φ ∈ SpE, since φ is an A-automorphism of E then $φ(e)^U$ = {U; φ(e_1), φ(e_2), ..., φ(e_{2n})} is also a local gauge of E. From the definition of the ricci curvature A-tensor given above and the relation σ (φ, φ) = σ, we can write

$Ric'_{|U}$ (φ(s), φ(t)) = ric'(φ(s), φ(t))
= $\sum_{i=1}^{2n} σ$(R'(φ(e_i), φ(s)) φ(t), φ(J (e_i))
= $\sum_{i=1}^{2n} σ$((φ R(e_i , s)t), φ(J (e_i))
= $\sum_{i=1}^{2n} σ$(R(e_i , s)t), J (e_i))
= ric(s, t)
= $ric_{|U}$ (s, t).

Now, we still recall that the symplectic Ricci operator is a section of EndE.Since φ ∈ SpE and R'(φ(s), φ(t))φ(r) = φ(R(s, t)r), for any s ∈ E(U) we can write

sRic' (φ(s))= $\sum_{i=1}^{n} R'$(φ(e_i), φ(J e_i))φ(s)
= ϕ($\sum_{i=1}^{n} R$(e_i , J e_i))s

= φ(sRic(s)). (19)

Proposition 3.2 Let {(E,), σ} be a symplectic Yang-Mills eld on X, the symplectic Ricci A-tensor is invariant under the action of the group sheaf of symplectomorphisms of E.

Proof. From the de nition of the symplectic Ricci A-tensor and the relation established above sRic' (φ(s)) = φ(sRic(s)) for any s ∈ E(U), we get

sric' (φ(s), φ(t)) = σ(sRic '(φ(s), φ(t)))
= σ(φ(sRic(s)), φ(t))
= σ(sRic(s), t)
= sric(s, t).

DECOMPOSITION OF A SYMPLECTIC CURVATURE A-TENSOR

De nition 4.1 Let (E, σ) be a symplectic vector sheaf on a topological space X,

T ∈ ^{4}E = Hom$_A$ (^{4}E, A) which satis es the following relations

- T (s, t, r, l) = T (t, s, r, l),
- T (s, t, r, l) = T (s, t, l, r),
- T (s, t, r, l) + T (t, r, s, l) + T (r, s, t, l) = 0,

for any s,t,r,l ∈ E(U) and an open subset U of X, is called a symplectic curvature A-tensor of E.

We denote by S(^{4}E*) the space of symplectic curvature A-tensors of E. The group sheaf of A-symplectomorphisms of E acts on S(^{4}E*) as follows:

SpE × S(^{4}E) → S(^{4}E)
(ϕ, T) → ϕ.T ,

for any s, t, r, l ∈ E(U) and an open subset U of X,

φ.T (s, t, r, l) = T (φ(s), φ(t), φ(r), φ(l)). (20)

Based on the classical patterns, see for instance (8), we get the direct sum decomposition

S(^{4}E) = S^0 (^{4}E) ⊕ S^r(^{4}E),

where S^0(^{4}E) is the subspace of symplectic curvature A-tensors T^0 with vanishing Ricci curvature A-tensor and S^r (^{4}E*) is the subspace of reducible T^r such that for any s, t, r, l ∈ E(U) and an open subset U of X,

T ^{r}(s, t, r, l) = 2σ(s, t)K(r, l) + σ(s, r)K(t, l) + σ(s, l)K(t, r) σ(t, r)K(s, l) σ(t, l)K(s, r)

where K ∈ ^{2}E is a covariant A-tensor symmetric of order 2 over E.

For a given T ∈ S(^{4}E), there exists T ^{0} ∈ S(^{o}E) and T^r ∈ S(^{r}E)

such that T = T^0 ⊕ T^r.

Hence, as sR belongs to S(^{4}E), this decomposition is sR = sR^0 + sR^r with sR^0 ∈ S^0 (^{4}E) and sR^r ∈ S^r(^{4}E) such that

sRr (s, t, r, l)= - $\frac{1}{2(n+1)}${2σ(s, t)ric(r, l) + σ(s, r)ric(t, l) + σ(s, l) ric(t, r) σ(t, r) ric(s, l) σ(t, l) ric (s, r)},

for any s, t, r, l ∈ E(U) and an open subset U of X, see (8, p.308).

De nition 4.2 Let (E, σ) be a symplectic vector sheaf on a topological space X, a symplectic A-connection on E is Ricci- at if sR^r = 0.

De nition 4.3 Let (E, σ) be a symplectic vector sheaf on a topological space X, a symplectic A-connection on E is of Ricci-type if sR^0 = 0.

Proposition 4.4 Let (E, σ) be a symplectic vector sheaf on a topological space X, the subspace of symplectic curvature A-tensor with vanishing Ricci curvature A-tensor is invariant under the action of the group sheaf of symplectomorphisms of E.

Proof. Let T ∈ S^0 (^{4}E), φ.T (s, t, r, l) = T (φ(s), φ(t), φ(r), φ(l)) implies that φ.T ∈ S^0(^{4}E*), see (20). For any s, t, r, l ∈ E (U) and an open subset U of X, by means of (16) and φ ∈ SpE one gets

φ.T (s, t, r, l) = T (φ(s), φ(t), φ(r), φ(l))
= σ(R(φ(s), φ(t))φ(r), φ(l))
= σ(φ(R(s, t)r, φ(l))
= σ(R(s, t)r, l)
= T (s, t, r, l).

Thus, φ.T = T .

Proposition 4.5 Let (E, σ) be a symplectic vector sheaf on a topological space X, the subspace of reducible symplectic curvature A-tensor is invariant under the action of the group sheaf of symplectomorphisms of E.

Proof. It follows from σ (φ, φ) = φ and (18).

PREFERRED SYMPLECTIC A-CONNECTIONS

We recall that for a given symplectic vector sheaf (E, σ) on a topological space X, a symplectic A-connection is a critical point of the functional

$$SYM( ) = \frac{1}{2}\int_X \sigma(R(\nabla), R(\nabla)) \frac{\omega^n}{n!} \quad (21)$$

if only if is a solution of the symplectic Yang-Mills equations

$^2{}_{SpE}$ (R( )) = 0, $\delta^2{}_{SpE}$ (R( )) = 0. (22)

De nition 5.1 Let (E, σ) be a symplectic vector sheaf on a topological space X i.e a locally free A-module of nite rank 2n over X endowed with a symplectic A-form σ, the preferred symplectic A-connections are the critical points of the symplectic Yang-Mills functional SYM.

Some authors use the following functional to de ne the preferred symplectic A-connections:

SYM': $Conn_A$ (E, σ) → A (23)

such that for any ∈ $Conn_A$ (E, σ)

$$SYM'( ) = \frac{1}{2}\int_X \sigma(sRic, sRic)\ \frac{\omega^n}{n!} \quad (24)$$

with (E, σ) a symplectic vector sheaf on a symplectic space (X, ω) and sRic the symplectic curvature A-tensor (see (Habermann, 2007)). The set of all preferred symplectic A-connections on (E, σ), denoted $Conn_A$ (E, σ)$_{SYM}$, is a subspace of $Conn_A$ (E, σ).

Let us now consider SpE × $Conn_A$ (E, σ)$_{SYM}$ → $Conn_A$ (E, σ)$_{SYM}$, the restriction of the action of SpE on $Conn_A$ (E, σ), (φ,) → φ φ ^{1} . This action de nes an equivalence relation on $Conn_A$ (E, σ)$_{SYM}$ and the quotient $Conn_A$ (E, σ)$_{SYM}$ /SpE is the moduli space of the preferred symplectic A-connections on (E, σ). A element of $Conn_A$ (E, σ)$_{SYM}$ /SpE is an orbit of a preferred symplectic A-connection.

De nition 5.2 Let (E, σ) be a symplectic vector sheaf on a topological space X and ric the Ricci curvature A-tensor of a symplectic A-connection , the equations

(s)ric(t, r) + (t)ric(r, s) + (r)ric(s, t) = 0 (25)

are called the Euler-Lagrange equations for the functional SYM, with s, t, r in E(U) and U an open subset of X. For symplectic manifolds, see (Boubel, 2003) page 748.

Proposition 5.3 Let (E, σ) be a symplectic vector sheaf on X, if a preferred symplectic A-connection then any element of the orbit of is also a preferred symplectic A-connection.

Proof. Let ( ) be the orbit of , for any '∈ ( ) there exists φ ∈ SpE such that ' = φ φ^{-1} (= φ^{-1} ' φ). Since is a preferred symplectic A-connection on E, the Euler-Lagrange equation holds i.e for any s, t, r in E(U), (s)ric(t, r) + (t)ric(r, s) + (r)ric(s, t) = 0. Replacing in the previous equation by φ ^{1} φ and using the invariancy of the Ricci curvature A-tensor under A-symplectomorphisms of E i.e ric(s, t) = ric'(φ(s), φ(t)), one obtains

'(φ(s))ric'(φ(t), φ(r))+ ' (φ(t))ric'(φ(r), φ(s))+ ' (φ(r))ric' (φ(s), φ(t)) = 0

Thus, 'is a preferred symplectic A-connection of E.

We remark that the Euler-Lagrange equation is invariant under the action group sheaf of A-symplectomorphisms of E.

Proposition 5.4 Let (E, σ) be a symplectic vector sheaf on a topological space X, if a Ricci- at A-connection then is a preferred symplectic A-connection.

Proof. Consider the decomposition $sR = sR^0 + sR^r$ of the symplectic curvature A-tensor. Since is a Ricci- at A-connection, $sR^r = 0$ i.e

sR^r (s, t, r, l) = - $\frac{1}{2n+1}$ {2σ(s, t)ric(r, l) + σ(s, r)ric(t, l) +σ(s, l)ric(t, r) σ(t, r)ric(s, l) σ(t, l)ric(s, r)}, for any s, t, r, l ∈ E(U) and an open subset U of X, see (6 p.308). It follows that the Ricci curvature A-tensor ric = 0 and is a solution of the Euler-Lagrange equation.

Proposition 5.5 Let (E, σ) be a symplectic vector sheaf on a topological space X, a symplectic A-connection with a Ricci curvature A-tensor is a preferred symplectic A-connection.

Proof. Since the Ricci curvature A-tensor is parallel i.e ric = 0, it is obvious that the Euler-Lagrange equation holds.

Conclusion

The main result of our paper consists in showing that the invariance of a symplectic A-connection relative to the group sheaf of symplectomorphisms on a symplectic vector sheaf implies the invariance of:

- the symplectic curvature A-tensor,
- the Ricci curvature A-tensor,
- the symplectic Riccci operator,
- the symplectic Ricci A-tensor
- under the group sheaf of symplectomorphisms.

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